

PAPER • OPEN ACCESS

## Effect of Increased Damping in Subordinate Oscillator Arrays

To cite this article: John Sterling *et al* 2018 *J. Phys.: Conf. Ser.* **1149** 012006

View the [article online](#) for updates and enhancements.

### You may also like

- [Fatigue crack detection in metallic structures with Lamb waves and 3D laser vibrometry](#)

W J Staszewski, B C Lee and R Traynor

- [Exploiting continuous scanning laser Doppler vibrometry \(CSLDV\) in time domain correlation methods for noise source identification](#)

Paolo Chiariotti, Milena Martarelli and Gian Marco Revel

- [Structural health monitoring using scanning laser vibrometry: III. Lamb waves for fatigue crack detection](#)

W H Leong, W J Staszewski, B C Lee et al.



The Electrochemical Society  
Advancing solid state & electrochemical science & technology

243rd ECS Meeting with SOFC-XVIII

**More than 50 symposia are available!**

Present your research and accelerate science

Boston, MA • May 28 – June 2, 2023

[Learn more and submit!](#)

# Effect of Increased Damping in Subordinate Oscillator Arrays

John Sterling<sup>1</sup>, Joseph Vignola, Ph.D<sup>2</sup>, Jenna Gietl<sup>2</sup>, Teresa Ryan<sup>3</sup>, Ph.D<sup>3</sup>, Noah Sonne<sup>3</sup>, Sai Tej Paruchuri<sup>4</sup>

<sup>1</sup>Naval Surface Warfare Center, Carderock Division  
9500 Macarthur Blvd, Bethesda, MD 20817

<sup>2</sup>Center for Acoustics, Vibrations and Structures  
Department of Mechanical Engineering  
The Catholic University of America  
Washington, D.C. 20064

<sup>3</sup>East Carolina University, Acoustics and Vibrations Laboratory  
Greenville, NC 27858

<sup>4</sup>Department of Mechanical Engineering  
Virginia Polytechnic Institute and State University  
Blacksburg, VA 24061

E-mail: [jsterling@gmail.com](mailto:jsterling@gmail.com)

**Abstract.** Previous literature has shown that large structures with a significant number of smaller attached structures exhibit much higher apparent damping than was predicted by models. Further research into this effect led to the discovery that arrays of such small attachments can be designed to alter the response of the overall system. That work also showed that small variation in the distribution of attachment mass or stiffness degrade the performance of the system. Additive manufacturing has now matured such that several methods provide the capability to realize the tolerances required to effectively test these designs. This paper discusses the use of StereoLithography Apparatus (SLA) optical fabrication to test the error sensitivity of different materials using laser vibrometry to characterize the mechanical behavior of the system.

## 1. Introduction

Previous works have examined the vibration response of structures with several smaller structures attached and found that the primary structure would often appear more damped than classical theory would have predicted. This phenomenon of apparent damping has been a topic of focused research for some time, such that alteration of a system's resonant response with smaller resonant arrays has been well characterized[1, 2, 3, 4]. Such sets of smaller structures have been termed a subordinate oscillator array (SOA). An SOA can be designed to effect specific response modifications in time or frequency domain[5]. This work examines the application in which the SOA is designed to remove energy in a particular frequency band from the primary structure and move the energy into the subordinate oscillators to dissipate within the subsystem. It has also been shown that small levels of disorder can drastically impact the response of the system. However, because metals have been the only feasible manufacturing option, they were the only materials considered in simulations thus far. Since the 3D printed materials considered



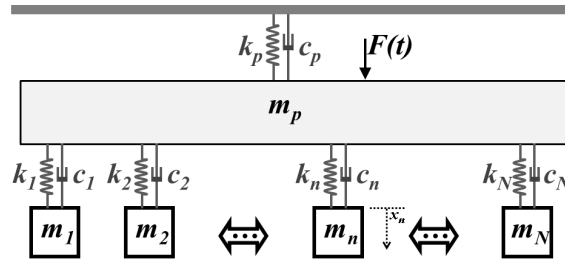


Figure 1:  $N + 1$  degree of freedom model of a mechanical system with  $N$  subordinate elements attached to the primary element. Each element has a distinct mass  $m_n$ , stiffness  $k_n$ , and damping  $c_n$ .

in this work are plastic, the quality factor( $Q$ ), which is an indicator of damping, has been measured to be approximately an order of magnitude less than that of typical metals. The lower  $Q$  results in a more disorder-tolerant system. In this work, experiment and simulation will be put forth showing how the lower  $Q$  can overcome the sensitivity to disorder, such as the disorder resulting from manufacturing variation. This result allows consideration of more manufacturing techniques for the production of the subordinate oscillator arrays.

The lumped element model is shown in Figure 1. This paper considers only the frequency response of the system. Figure 2 shows several configurations of the system (Figure 2a) and one configuration with increasing amounts of added disorder (Figure 2b). For this paper, disorder is defined as a statistical average of deviation errors in individual oscillators.

Figure 2a includes four different distributions of the masses,  $m_n$ , stiffnesses,  $k_n$  with  $Q$  of 62.5. Selection of the property distributions allows for tuning of system bandwidth, as illustrated here with bandwidths ranging from 1.25% to 10%. The 10% disorder case from Figure 2a is used as an example in Figure 2b. Statistically inducing increasing levels of disorder (from 0.01% to 10%) into the frequency distribution of the oscillators shows how little disorder is required to degrade the response of the system. Figure 2b illustrates this progressively increasing effect of fabrication error on a bandpass response. This work examines results for the 3D printed resin with a  $Q$  of 14.1 in both analytical and experimental situations.

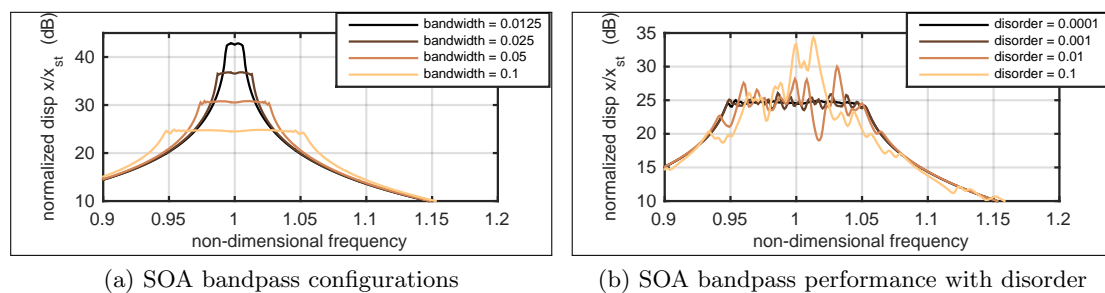


Figure 2: (2a) The array of attachments can be designed to make the response of the primary element behave as a bandpass filter (2b) A bandpass response was designed to have a 10% bandwidth. The performance degrades as error in the property distribution increases from one part in  $10^4$ , where the bandpass performance is unaffected, to 10% where the bandpass is response is not apparent.

## 2. Design Parameters of a Subordinate Oscillator Array

Previous work[5] has focused on simplifying the variables required to describe the SOA analytically. Instead of describing specific length, width, and thickness dimensions for each element, the array is specified according to distributions of frequencies and masses across  $N$  oscillators. In this method, design of effective bandpass arrays is simplified as the frequency distribution combines length and thickness and the mass ratio (defined as a ratio of the sum of masses of the individual elements to that of the primary mass). The mass ratio can be adjusted by changing subordinate cantilever width without an effect on the frequency, as long as the aspect ratio of the subordinate cantilever is maintained to be beam-like and not move to a more plate-like regime. The frequency distribution minimums, maximums, and frequency spacing define the band and shape of the effect. The mass distribution contributes to the magnitude of the bandpass response as well as edge effects. The comprehensive effects of the mass distribution are still under investigation.

Additional work [6] puts forward a simple equation relating array parameters that can predict the minimum number of elements required to produce a flat bandpass response in the primary. The minimum number of oscillators is determined by

$$N = \eta Q \Delta$$

where  $\eta$  is the modal overlap,  $Q$  is quality factor and  $\Delta$  is the desired fractional bandwidth. The fractional bandwidth can be easily defined as the width of the filter effect. Modal overlap is a measure of how densely packed the modes are, calculated by half-power bandwidth divided by the  $\Delta f$  between peak frequencies. Modal overlap,  $\eta$ , must be greater or equal to 2 for the filter to function correctly[6]. With modal overlap  $\eta > 2$ , it is predicted to increase the robustness of the design and decrease disorder sensitivity.

## 3. Effects of Quality Factor on Bandpass Performance

When considering the requirements to design a bandpass response, it follows easily that a design using subordinate cantilevers with lower  $Q$  would require a lower  $N$  (fewer cantilevers) due to the modal overlap requirement. With wider individual peaks, a given band can be sufficiently covered with fewer elements in the subordinate array. What had not been predicted was how lower  $Q$  materials would significantly reduce the disorder sensitivity of the overall response because the response of the oscillators were no longer so highly localized in the frequency domain. Figure 2b shows that with even moderate disorder, errors in individual elements can cause undesirable local behavior in the response of the system, disrupting the desired flat band response. In that higher  $Q$  situation, slight geometric error in even a single cantilever can cause the overall dB reduction in the band to be greatly reduced.

The desire to decrease the quality factor and also the cost of production leads 3D printed plastics to become an attractive option. Metal waterjet machining costs were in the hundreds of dollars each for previous prototypes and these 3D printed prototypes cost dollars each. In addition, in many applications, the use of 3D printed material still requires some element of compromise on some preferred property of the printed component. Those compromises are made in deference to the advantage of speed or design flexibility that 3D printing offers. In the bandpass application of the SOA, given the demonstration that a lower  $Q$  material has a lower sensitivity to disorder, the 3D printed component is actually the preferred option.

## 4. Specimen Production

Prior simulation work focused on metals, and with  $Q$  values of 100 or more, the metal designs required numerous oscillators. As such, error in any individual oscillator could have a significant impact on the overall response. In the era of rapidly developing 3D printing technology,

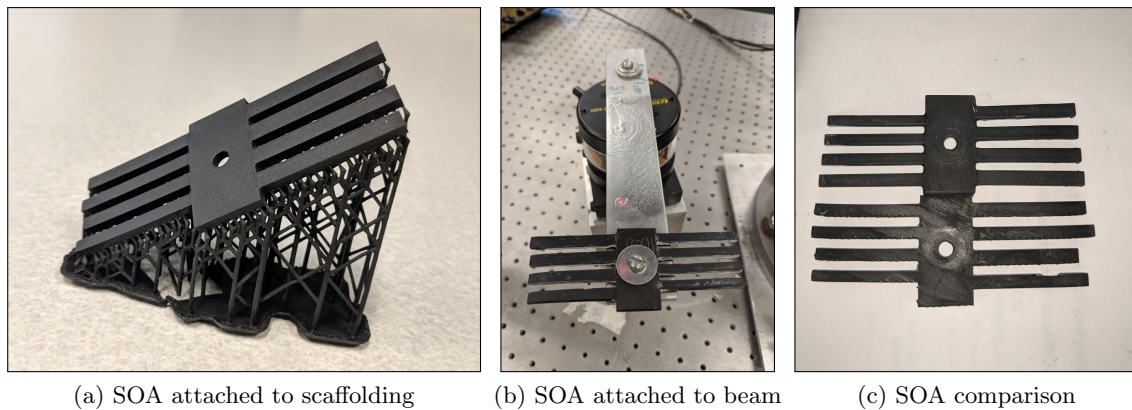


Figure 3: (3a) A depiction of the SOA as printed by the Form 2 printer (3b) Shows the 10% percent SOA as attached to the beam and shaker. (3c) This is a comparative photo of the 10% SOA and the as-designed SOA one.

stereolithography apparatus printers, using lasers to harden resin, are able to construct parts meeting the tolerances that were formerly only available in a machine shop. The Form 2 SLA printer from FormLabs used for these prints has a smallest repeatable feature size of 150 microns. This is comparable to the performance of professional machine shops and makes disorder levels of 1/500 achievable even with these small parts. Figure 3a shows the final product of the 3D print process. The FormLabs GPBK-03 resin has a measured  $Q$  of 14.1 after a 30 minute IPA wash in the Form Wash machine and a 60 minute cure at 50 C in the Form Cure machine.

### 5. Laser Vibrometry Setup

A custom scanning laser Doppler vibrometer system, developed at Catholic University of America and thoroughly described in [7], is used to characterize the printed arrays. The system consists of a Polytec OFV-5000 vibrometer controller and a Polytec OFV-501 remote head. A LABVIEW generated chirp excitation is routed through a Krohn-Hite 7400 amplifier and into a Pasco 9324 shaker.

The simplest implementation of the SOA system is exhibited here as a primary beam on the shaker with an SOA mounted. Figure 3b shows the SOA system affixed to the primary beam and the laser measurement spot on the primary beam.

### 6. Experimental Results of Increased Damping

With the increased damping of SLA resin, the requirement of precise frequency distribution ordering is greatly relaxed without the possibility of a single, high- $Q$  oscillator causing local disturbances. While the tolerances for the metal SOAs were almost impossibly tight, Figure 3c and Figure 5a show that even though there is a higher minimum disorder in the 3D printed parts, the results still shows a 17 dB peak reduction in this idealized situation. There is an offset in the center frequency response of the SDOF system from the SOA response systems due to the lack of accounting for the plate of material that connects the oscillators. Future work will account for that in order to properly match the center frequency of the SOA to that of the primary mass.

Prior results with low disorder metal samples in Figure 4a demonstrated less in-band ripple as shown in Figure 4b. Also evident is the fact that built in randomized disorder at one percent destroyed the broadband nature of the suppression. This is visible in how the 10% disorder case

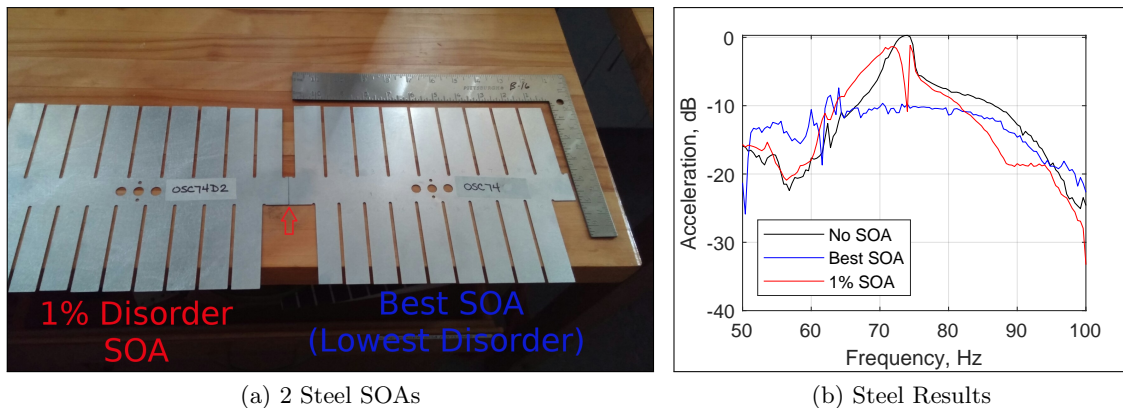


Figure 4: (4a) This shows the steel 1% disorder and as-designed SOA side by side. Note how much closer in length the individual cantilevers are compared to the plastic ones. There is a small gap between the two pieces in the center with an arrow pointing at it for clarity. This shows how important the lower  $Q$  is for disorder tolerance (4b) This shows the initial disorder test response with steel SOAs. Note the low disorder curve works in a nearly ideal fashion, while the notch-filter curve has this response from a built-in 1% disorder

(orange curve) of Figure 5b provides a better reduction than the 1% disorder case (red curve) of Figure 4b.

These results show that SOAs with this high level of damping are effective and likely preferable in most situations as the low  $Q$  system with 10% disorder mitigates the hypersensitivity to disorder better than the 1% disorder displayed by the metals. This hypersensitivity was the limiting factor in the ability to implement these in real-world situations. These results also show a strong agreement between the MATLAB model and the experimental result (Figure 5).

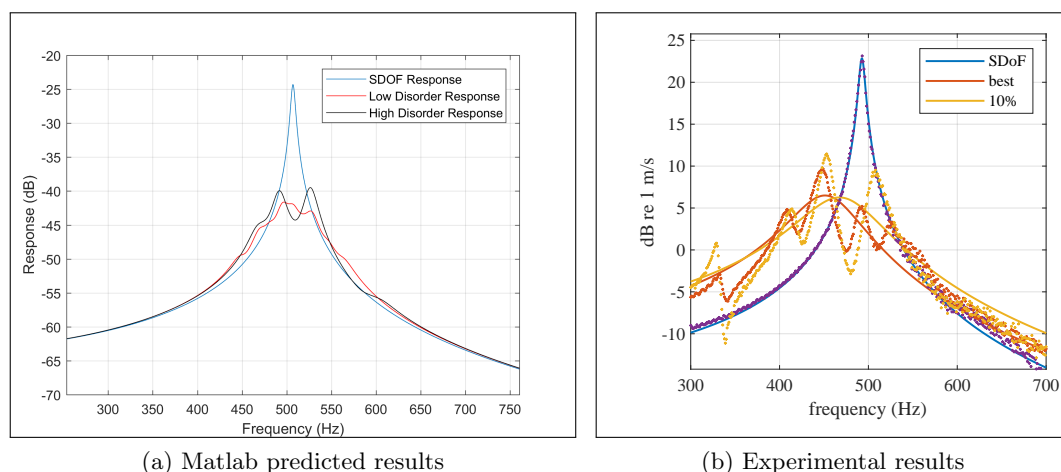


Figure 5: (5a) This plot shows the idealized Matlab simulation of the low and high disorder SOAs pictured in Figure 3c (5b) This plot shows the experimental results of the low and high disorder SOAs using 3d-printed plastic. The dots represent the experimental data and the solid lines are a curve fit using half-power bandwidth, center frequency and  $Q$  of the data.

## 7. Conclusions

Early work with metal, high- $Q$  SOAs showed an extreme sensitivity to error in the frequency distribution of the isolated natural frequencies of the oscillators. In such high- $Q$  cases, error ratios of 1/1000 have a noticeable effect and 1/100 have significant effect. In contrast, the 10% low- $Q$  disorder SOA was well within acceptable margins compared to the as-built SOA given the expected ranges of variation that may occur due to manufacturing variation, fatigue, or degradation over time. Use of low  $Q$  materials from 3D printers significantly expand the design space for the SOA. The lowered sensitivity of the arrays manufactured with plastic materials reduces production time and cost, and the arrays perform with equivalent or better metrics.

The 10%, low- $Q$  disorder SOA had a nearly equivalent response to the as-designed low  $Q$  SOA at this damping level. This means an as-designed SOA has a high probability of success in the field in keeping its designed damping capabilities over time. This has prompted a future investigation into the feasible design space for the SOA. It also prompted investigation of a number of materials and manufacturing methods that were not previously considered as feasible alternatives. This is especially helpful for other, more traditional 3D printing methods which use nozzles instead of laser-hardened resin. In future work, the capabilities of different 3D printing methods and materials will be investigated and a design space will be created to inform future SOA manufacturing methods.

## References

- [1] L. Kitis, B.P. Wang, and W.D. Pilkey. Vibration reduction over a frequency range. *Journal of Sound and Vibration*, 89(4):559–569, August 1983.
- [2] M. Strasberg and D. Feit. Vibration damping of large structures induced by attached small resonant structures. *J. Acoust. Soc. Am.*, 99(1):335–344, 1996.
- [3] G. Maidanik. Induced damping by a nearly continuous distribution of nearly undamped oscillators: linear analysis. *Journal of Sound and Vibration*, 240:717–731, 2001.
- [4] A. Akay, Z. Xu, A. Carcaterra, and I. Murat Ko. Experiments on vibration absorption using energy sinks. *J. Acoust. Soc. Am.*, 118:3043–3049, 2005.
- [5] J.F. Vignola, J.A. Judge, and A.J. Kurdila. Shaping of a system's frequency response using an array of subordinate oscillators. *Journal of the Acoustical Society of America*, 126:129–139, 2009.
- [6] Joseph Vignola, Aldo Glean, John Judge, and Teresa Ryan. Optimal apparent damping as a function of the bandwidth of an array of vibration absorbers. *The Journal of the Acoustical Society of America*, 134(2):1067–1070, August 2013.
- [7] P. O'Malley, T. Woods, J. Judge, and J. Vignola. Five-axis scanning laser vibrometry for three-dimensional measurements of non-planar surfaces. *Meas. Sci. Technol.*, 20(11):115901, 2009.